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A new method for improving the interferometric resolution by compensating for the atmospherically induced phase shift

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Abstract. Electromagnetic waves traveling through the atmosphere experience an added phase shift due to the presence of various gases. Since the water vapor distribution in the atmosphere has a significant spatial and temporal variance, the corresponding phase shift can cause a serious degradation of the maps produced by radio interferometers. This study presents the calculation of the atmospherically induced phase shift in terms of ground based measurements and introduces a new technique to compensate for it. The compensation technique is implemented on the Berkeley-Maryland-Illinois Array at Hat Creek, California, and various tests are performed to demonstrate the accuracy of the technique. Finally, some future work is suggested.

1. Radio Interferometry With Atmospheric Effects

Interstellar space consists primarily of a vacuum through which optical and radio signals pass with minimal degradation (the signals can be absorbed and scattered by interstellar dust). Earth's atmosphere, however, is composed of various gases, including water vapor, that collectively act to degrade the signals that ultimately reach the radio telescopes on Earth.

The atmospheric degradation of radio signals is due to two conditions. First, the amplitude of radio signals is attenuated due to losses in the dielectric (the atmosphere) through which they pass. Second, the phase of the signal is a function of the path through which it passes, and the atmospherically induced phase shift (AtIPS) experienced by the signal will be a function of the distribution of the atmospheric constituents in its path. When the frequencies of interest are not in the vicinity of any atmospheric resonant lines, the decrease in amplitude has far less impact on the resulting radio maps than the AtIPS introduced due to the presence of the atmosphere.

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The radio interferometer output is the product of the amplitudes and the difference of the phases (this phase difference is called the interferometer phase) of the signals received by the individual antennas. (See *Thompson et al.* [1986] for a discussion of radio interferometry.) So, if the AtIPS, no matter how large, were the same for each signal path, the atmospheric effects on the interferometer phase would cancel and therefore would have no effect on the final result. However, this is not typically the case in actual measurements. Although water vapor is not the largest contributor to the atmospherically induced phase shift, the integral of the water vapor content is very much a function of the atmospheric path. And because the signal to each antenna passes through a different path in the atmosphere, the AtIPS difference experienced by each signal due to the water vapor will be much larger than the AtIPS difference due to the other atmospheric gases. The farther apart the antennas are, the more different the individual AtIPS will be.

Since the accuracy of calculations in radio interferometry strongly depends on the phase accuracy, this change in phase will degrade features of the map, add extra noise, and impose limitations on the resolution of interferometric observations. The quality of the radio maps and the utility of radio interferometers in general could be significantly improved if the atmospheric effects were fully (or at least partially) compensated.

The fundamental limitation that the AtIPS places on the radio interferometer is this: The antennas must be kept close enough to one another to minimize the degradation introduced by the AtIPS, but far enough apart to obtain sufficient map resolution, since the interferometric resolution is inversely proportional to the antenna separation.

The effect becomes especially severe in the summer, when the air is turbulent and humid. During this time the antennas must be close together in order to minimize the AtIPS difference and obtain a useful radio map. If the effects of the atmosphere could be compensated for, then the distance between the antennas could be increased, with a consequent improvement in the quality of the radio maps.

This work presents a new technique to correct the AtIPS experienced by the signals received by a radio interferometer. It was developed and then implemented on the Berkeley-Illinois-Maryland Array (BIMA) at Hat Creek, California. This interferometer consists of three 6-m antennas and operates in the 72-115 GHz frequency range. This work is based on a Ph.D. thesis [*Zivanovic*, 1992] which has more details about the derivations presented in this paper.

2. Previously Used Methods for AtIPS Compensation

The problems introduced by the AtIPS were studied by radio astronomers for a number of years [*Baars*, 1967; *Hinder and Ryle* 1971; and *Liebe*, 1981], and several techniques have been developed to correct for these atmospheric effects. The most common technique used today is the observation of a "calibrator source." (This technique is now being used at the BIMA.) A calibrator source is any extraterrestrial radio source that is small in angular size and, hence, effectively a point source to the interferometer. The interferometer phase from the calibrator observation is equal to the sum of the geometrical delay (which can be removed) and the AtIPS. Therefore the AtIPS as a function of time can be obtained and subtracted from the data.

This technique has several shortcomings. First, the observation must be interrupted periodically in order to observe the calibrator. Second, the calibrator is not in the same angular location as the source under observation and is not observed at the same time. Therefore the signal travels through a different portion of the atmosphere, and the AtIPS added to the calibrator signal is different from the AtIPS added to the source signal. The difference between these two atmospheric phases is a function of the angular separation of the two extraterrestrial radio sources and the atmospheric conditions. (Since a limited number of calibrators exist, the angular separation between the source and calibrator can be as large as 45 degrees.) Other attempts to measure and compensate for the AtIPS are summarized below.

Baars [1967] used the Greenbank interferometer to observe a calibrator source for several hours and at the same time recorded meteorological data such as local temperature, pressure, and humidity. They observed a correlation between the expected phase fluctuations (calculated using the meteorological fluctuations) and the actual phase difference. Although no suggestions were made as to how these results can be used to compute a phase correction, this early experiment demonstrated that there is a correlation between atmospheric conditions and the phase shift experienced by signals traveling through the atmosphere.

A water vapor radiometer (WVR) designed and built at the Jet Propulsion Laboratory [*Westwater*, 1978; *Westwater and Guiraud*, 1980] was tested at the very large array (VLA) interferometer in New Mexico. Two WVRs operating at $f_1 = 20.7$ GHz and $f_2 = 31.4$ GHz were used. The operating frequencies were selected to provide a measurement at the half-power point of the 22 GHz transition line and in the valley of the water absorption spectrum. The radiometers were mounted on each antenna, and a calibrator source was observed. For short observations (up to 2 hours) the calibrator data and WVR measurements did have a good correlation, but, for longer observations, the WVRs exhibited a distinct drift, and eventually their outputs were completely dominated by the system instabilities.

A number of groups [*Elgered et al.,* 1982; *Guiraud et al.,* 1979; *Moran and Rosen,* 1981; *Rocken et al.,* 1991, *Westwater et al.*, 1990] used WVRs to measure the water vapor content along the signal path through the atmosphere. From these results they were able to obtain an estimate of the excess propagation path L_o introduced by the presence of the atmospheric water. In each case, though, the success was still very limited, primarily due to the instability of the WVRs.

The radiometer is, however, still used in a considerable number of interferometric observations to at least partially compensate for the atmospheric effects. In very long baseline interferometry (VLBI) observations the baselines are long enough that a radiometer mounted on each antenna is necessary to provide an initial estimate of the individual AtIPS. A calibrator source is then used for the final correction.

The early tests did show a correlation between atmospheric conditions and the AtIPS, but none of them resulted in a practical and efficient algorithm to determine a correction factor. In each case, the conditions necessary to obtain a good measurement of the AtIPS were very restrictive. For instance, the accuracy of the calibrator source is dependent on weather variability and the angular distance between the source and calibrator; the radiometer is only effective for short observations.

3. New Technique to Compensate for the AtIPS

Despite the fact [*Elgered*, 1993] that the excess propagation path due to the main atmospheric components, oxygen and nitrogen, can be fairly large (about 2.3 m at the zenith), it can be ignored since it has a very small spatial and temporal dependance. On the other hand, the excess propagation path from water vapor can vary unpredictably in both space and time, between a few millimeters (or less) and 40 cm in the zenith direction [*Elgered*, 1993]. Therefore the AtIPS due to the presence of water vapor in the atmosphere is the dominant component. In the next section the AtIPS due to the water vapor is calculated in some detail.

3.1. Calculation of the AtIPS

The phase shift ϕ due to the presence of water vapor in the atmosphere can be written as

$$\phi = \frac{2\pi}{\lambda} \int_{atm} (n_{ro} - 1) \frac{\rho_w(z)}{\rho_{wo}} \, dz \qquad (1)$$

where n_{ro} is the real part of the index of refraction of water vapor, λ is the wavelength, ρ_w is the water vapor density, ρ_{wo} is the density of water (1000 kg/m^3), and the integration is along the propagation path.

The refractive index of water vapor, over this desired frequency range (72-115 GHz), can be written as [*Meeks*, 1976]

$$n_{ro} = 1 + 7.76\times10^{-7} \frac{P_D}{T} + 6.48\times10^{-7} \frac{P_w}{T} + 3.776\times10^{-3} \frac{P_w}{T^2} \qquad (2)$$

where T is the absolute temperature in degrees Kelvin, P_D is the partial pressure of dry air in pascals, and P_w is the partial pressure of the water vapor in pascals. It can be shown that the total atmospherically induced phase shift ϕ experienced by the signal received by an antenna, is given by the following expression:

$$\phi = 10^{-3} \frac{2\pi}{\lambda} \left[0.228 \frac{P_o}{g} \sec\theta_o + \int_{atm} \left(0.076 + \frac{1742}{T(z)}\right) \rho_w(z) dz \right] \qquad (3)$$

where ρ_w is the water vapor density as a function of atmospheric height and antenna location, θ_o is the angle between the zenith and the antenna axis (pointing angle), g is the acceleration due to gravity, $T(z)$ is the atmospheric temperature as a function of altitude z, and P_o is the air pressure on the ground (MKS units are used). P_o has only a slight variation from antenna to antenna; therefore, the first term in equation (3) will have a negligible contribution to the interferometer phase and will be ignored.

If it is assumed that the temperature is relatively constant over the region where the water vapor density has significant values (roughly the first 4 km of the atmosphere), then the AtIPS can be written as

$$\phi = 10^{-3}\frac{2\pi}{\lambda}\left[0.076 + \frac{1742}{T}\right]\int_{atm} \rho_w dz \qquad (4)$$

where T is the temperature on the ground and can be measured. This assumption of constant temperature is certainly justified in the Hat Creek area. The remaining quantity, the precipitable water vapor (PWV), $\int \rho_w dz$, can be obtained from the expression for the total signal power.

The total power of the signal received by one antenna, P_T, is given by

$$P_T = k\Delta f\,[\,T_{sB}\,e^{-\tau_f} + T_R + T_{sc} + \int_{atm} T(z)\,\kappa_f\,e^{-\tau_f(z)}\,dz] \qquad (5)$$

where T_{sB} is the brightness temperature emitted by the extraterrestrial radio source under observation, κ_f is the absorption coefficient of water vapor, τ_f is the optical depth, T_R is the receiver temperature, T_{sc} is a measure of the ground scattering picked up by the antenna, k is the Boltzmann constant, and Δf is the bandwidth of the interferometer (830 MHz for the BIMA system).

It is assumed that the extraterrestrial radio source emission is much weaker than the atmospheric emission, and that the optical depth is small. This is a good assumption for the BIMA system, since the typical source power is several orders of magnitude smaller than the atmospheric emission. Furthermore, if it is again assumed that the temperature is reasonably constant over regions in the atmosphere where the water vapor density has significant values, then the total signal power can be written as

$$P_T = k\Delta f\left[T_R + T_{sc} + T\kappa_f\int_{atm} \rho_w dz\right] \qquad (6)$$

T_R and T_{sc} can be measured for each antenna of the BIMA [*Zivanovic*, 1992] and then subtracted from the total received power to obtain $k\Delta f T_w$, the power due to the atmospheric water vapor emission. Now one can solve for the PWV from (6), and substitute it in (4) to get

$$\phi = 10^{-3}\frac{2\pi}{\lambda}\left[0.076 + \frac{1742}{T}\right]\frac{T_w}{T\kappa_f} \qquad (7)$$

where

$$T_w \equiv P_T/k\Delta f - (T_R + T_{sc}) \qquad (8)$$

An estimate of the error introduced by assuming constant temperature in the atmosphere is illustrated in Figure 1. This figure represents a comparison of the ratio of ϕ and T_w calculated using the exact expressions (equations (1) and (3)), with the ratio calculated using equation (7) (approximation). The temperature as a function of atmospheric height (needed to evaluate the exact expressions) was taken from United States Standard Atmosphere (1962). The water vapor density as a function of atmospheric height was taken to be a decaying exponential with a scale height of 2 km [*Meeks*, 1976]. The ground temperature used in these calculations was 287 K.

The error introduced with the approximation is tolerable since the use of the technique will definitely improve the phase measurements. Therefore the atmospherically induced phase shift can be calculated from ground-based measurements.

3.2. Implementing the New Technique in the Berkeley-Illinois-Maryland Array

A total power detector, which samples the output signal, was added to the new IF amplifier. This system was designed and built for the BIMA to measure the total signal power P_T with

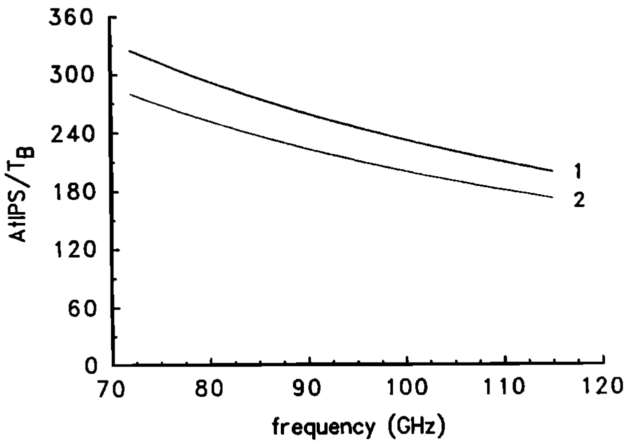

Figure 1. Comparison of ϕ/T_w calculated with and without the assumption that the atmospheric temperature is constant in regions where the water vapor density has significant values. The temperature on the ground is 287 K and the water vapor density on the ground is 5×10^{-3} kg/m^3 . Curve 1, calculation with approximation; and curve 2, calculation without approximation.

high accuracy and thermal stability. The thermal stability of the circuit was selected such that the total power fluctuations ΔP_T due to the changes in the circuit temperature are

$$\frac{\Delta P_T}{P_T} \leq 10^{-3}$$

This is because the smallest power increment (as a fraction of the average power) that can be measured by the current-sampling and power-measuring system at BIMA is about 10^{-3}.

Three such amplifiers (with power detectors) were built and installed in the BIMA, and the following measurements were performed to calibrate the system: (1) The proportionality constant between the output of the total power detector (in volts) and the total signal power (in kelvins) was determined; (2) the receiver temperature T_R was measured; and (3) the antenna ground pickup T_{sc} as a function of the antenna pointing angle and azimuthal angle was measured. T_R and T_{sc} are subtracted from the total power detector measurements to obtain T_w , and the AtIPS is calculated using equation (7).

Before each observation some additional calibration is performed. The zero level of the total power detector and the proportionality constant necessary to convert the analog to digital (A/D) output (in volts) and power (in kelvins) are measured. Then a calibrator source is observed for several minutes to measure the proportionality constant between the total power and the AtIPS. This gives a more accurate value for the proportionality constant than the value predicted by the theory.

The correction can be applied directly to each signal during the observation, as follows: At the beginning of each integration cycle the atmospheric phase is computed using the total power measurements and added to the phase of the first local oscillator. Every 0.3 s the change in the total power (from the preceding value) is measured and the resulting change in phase added to the local oscillator phase.

This new technique has the advantage that the AtIPS is measured continuously and along the signal path (instead of along a different path as with the calibrator source technique). The correction can then be applied directly to the signal, in real time, before any data processing has taken place. Finally, longer baselines can be used with this correction technique, thereby improving the resolution.

4. Testing the Correlation

To demonstrate the correlation between the AtIPS and the total signal power, a calibrator source was observed, and the resulting interferometer phase was compared to the total signal power differences measured during the observation. A very short integration time, 0.3 s, was used so that the interferometer phase could be compared to the instantaneous total power measurements that were sampled at the same rate. The initial results contained a considerable amount of system noise. Instead of adding a low-pass filter to the interferometric system, the data were numerically filtered after the observation. This was done by convolving the data with the Fourier transform of a low-pass filter $Z_c(t)$, given by

$$Z_c(t) = \begin{cases} \dfrac{\sin 2\pi f_c t}{\pi t} & |t| \leq t_o \\ 0 & |t| > t_o \end{cases}$$

where $f_c = 0.075$ Hz and $t_o = 45$ s.

Results from several of these tests are shown in Figures 2-4. Each observation was in fairly clear weather; the results are for a baseline of 12 m. During the observation the sources were at high elevation angles (around the zenith) in order to minimize the unwanted ground scattering. In each case the zero level of the phase and total power do not match up, primarily due to calibration errors. The zero levels and gain of the total power detector were different for each observation and had to be calibrated each time.

Figure 2 is for a 5-min observation of Jupiter. The temperature during the observation was 281 K, so the predicted proportionality constant between the interferometer phase (measured in degrees) and the difference between the signal powers (measured in degrees Kelvin) is 202°/K. The measured proportionality constant is 185°/K.

Figure 3 is for a 4-min observation of Mars. The temperature during the observation was 278 K, so the expected proportionality constant is 197°/K. The measured proportionality constant is 186°/K.

Figure 4 is for a half-hour observation of Jupiter. After the numerical filtering was performed on the data, every three data points were averaged, so for this case, there is one data point every 0.9 s. The temperature during the observation was 279 K, so the expected proportionality constant is 208°/K. The measured proportionality constant is 197°/K. In this observation the total power seems to lag the phase at some times. This is primarily due to the large time constant in the total power detector and other system drifts.

These results indicate that there is indeed a definite correlation between the AtIPS and the total power. Much of the variance can be accounted for by the system noise, since the original interferometer was not designed to provide the sensitivity necessary for these experiments. A least squares fit of the sets of data was performed to compute an estimate of the AtIPS using the difference between the signal powers. This phase was then subtracted from the interferometer phase (which in this case is equal to the AtIPS). The resulting phase was typically less than the original measured phase (which in this case is equal to the AtIPS) by at least a factor of two.

It is expected that the full benefit of the new technique will only be realized when the new and improved components (for example, the receiver,

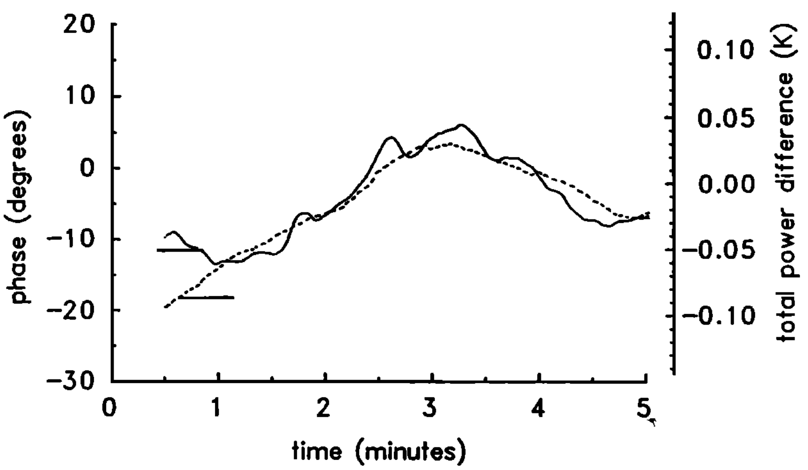

Figure 2. Observation 1: comparison of phase fluctuations with total power fluctuations for an observation of Jupiter on March 18, 1992. Observation was started at 2345 LT; temperature was 320 K.

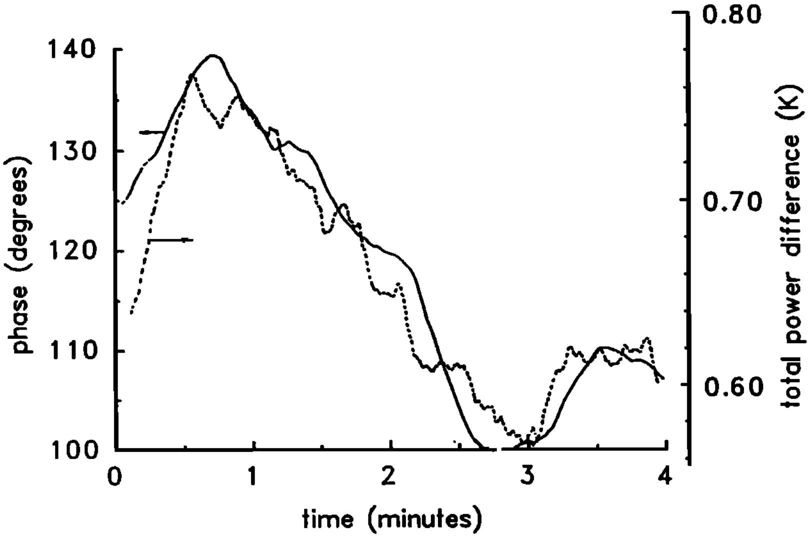

Figure 3. Observation 2: comparison of phase fluctuations with total power fluctuations for an observation of Mars on May 22, 1992. Observation was started at 1700 LT; temperature was 324 K.

antennas, and mixers) are installed in the BIMA. In the current system, the noise and instability of these components are too large to provide the high sensitivity required to realize the full benefit of the accurate AtIPS measurements. At present, the BIMA is being expanded to six antennas, and a majority of the components are being redesigned in order to decrease the noise figure of the entire system.

5. Conclusions and Future Work

The initial tests demonstrate that there is a definite correlation between the total signal power

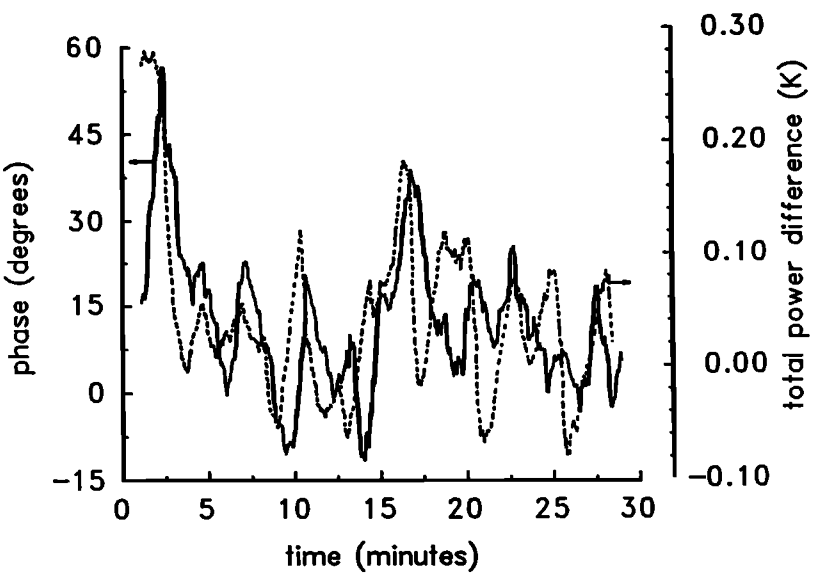

Figure 4. Observation 3: comparison of phase fluctuations with total power fluctuations for an observation of Jupiter on April 6, 1992. Observation was started at 2345 LT; temperature was 315 K.

and the atmospherically induced phase shift. Most of the observed differences between the measured phase and the power radiated by the atmosphere (water vapor) were due to system noise. In spite of the receiver noise, the usefulness of the technique has definitely been demonstrated on a practical case. It is expected that the new and improved BIMA will provide a better demonstration of this correlation and that this technique will be routinely used to correct for the atmospheric effects. Once the necessary proportionality constants have been accurately measured, the correction can be applied during the observation in real time. This improvement would not only increase the quality of the radio maps, but would also increase the utility of the interferometer, since longer baselines could then be used. The final test of the new technique will be to apply it to a complete observation of an extraterrestrial radio source using several different antenna configurations.

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